AD-A276 622

AGE

, including the time for reviewing instructions, searching existing data sources, gathering and
ts regarding this burden or any other aspect of this collection of information, including suggestions
1s and Reports, 1215 Jefferson Davis Highway, Suite 1204, Arlington, VA 22202-4302, and to
20503.

1. Agency Use C	April 1993	3. Report Type and Dates Covered. Final - Proceedings

4. Title and Subtitle.
The decomposition of guided waves into spherical representations and near-field effects on target scattering

5. Funding Numbers.
Program Element No. 0601153N
Project No. 3202
Task No. 340
Accession No. DN255011
Work Unit No. 571505703

6. Author(s).
M. F. Werby, G. V. Norton, and E. White

7. Performing Organization Name(s) and Address(es).
Naval Research Laboratory
Ocean Simulation and Tactics Branch
Stennis Space Center, MS 39529-5004

8. Performing Organization Report Number.
NRL/PP/7181--93-0037

9. Sponsoring/Monitoring Agency Name(s) and Address(es).
Naval Research Laboratory
Center for Environmental Acoustics
Stennis Space Center, MS 39529-5004

10. Sponsoring/Monitoring Agency Report Number.
NRL/PP/7181--93-0037

11. Supplementary Notes.
Published in SPIE.

94-06722

12a. Distribution/Availability Statement.
Approved for public release; distribution is unlimited.

DTIC
ELECTE
MAR 01 1994
S

13. Abstract *(Maximum 200 words).*

We wish to develop a scheme that will enable us to predict the near field that results from the interaction of a submerged object with a guided wave for a general object in a general waveguide. The method will be based on the extended boundary condition (EBC) method (T-matrix) of Waterman and a new normal mode method that allows one to decompose the normal-mode solutions into a spherical representation suitable for operating on the spherical tensors that arise from the T-matrix method. Of particular importance is the fact that the resulting formulation allows one to couple the resulting near-field solution to an outgoing normal-mode series that leads to the general waveguide solution.

94 2 28 117

94-06054

14. Subject Terms. Acoustic scattering, shallow water, waveguide propagation	15. Number of Pages. 11
	16. Price Code.

17. Security Classification of Report. Unclassified	18. Security Classification of This Page. Unclassified	19. Security Classification of Abstract. Unclassified	20. Limitation of Abstract. SAR

Best
Available
Copy

Automatic Object Recognition III

Firooz A. Sadjadi
Chair/Editor

14–16 April 1993
Orlando, Florida

Accesion For		
NTIS CRA&I		$\checkmark$
DTIC TAB		$\square$
Unannounced		$\square$
Justification		
By		
Distribution/		
Availability Codes		
Dist	Avail and/or Special	
A-1	20	

DTIC QUALITY INSPECTED 2

94 2 24 017

The decomposition of guided waves into spherical representations and near-field effects on target scattering

M. F. Werby, G. V. Norton, and E. White

Naval Research Laboratory
Theoretical Acoustics Branch
Stennis Space Center, MS 39529

ABSTRACT

We wish to develop a scheme that will enable us to predict the near field that results from the interaction of a submerged object with a guided wave for a general object in a general waveguide. The method will be based on the extended boundary condition (EBC) method (T-matrix) of Waterman and a new normal mode method that allows one to decompose the normal-mode solutions into a spherical representation suitable for operating on the spherical tensors that arise from the T-matrix method. Of particular importance is the fact that the resulting formulation allows one to couple the resulting near-field solution to an outgoing normal-mode series that leads to the general waveguide solution.

I. INTRODUCTION

Scattering by submerged objects in the ocean can be complicated by the sort of environment that they are in. The easiest problem for submerged targets is when one can treat the problem as if no boundaries exist. This can happen at high frequencies in deep oceans with targets reasonably near to the source. The problem becomes more difficult if one must include boundaries such as ocean surfaces and bottoms. Moreover, a refractive water column further complicates the problem both in the formal sense, as well as for computation. Here, we will focus on submerged objects that may have a variety of boundary conditions. The objects are allowed to be in a complicated waveguide, which can be described by normal mode theory, and for which we are only interested in the near-field solution. The general solution in a waveguide is presented by Norton et al.,[1] based on the near-field solution presented here. The approach is from its inception approximate, but we believe it is quite adequate and robust for most environments and targets.

Presently much progress has been made for targets in a free environment. The targets can range from spherical shapes or infinite cylinders to elongated targets. Indeed, fairly general targets are now being successfully treated by variations of Waterman's Extended Boundary Condition (EBC) method[2-5] for impenetrable, as well as fluid and elastic targets, particularly if they have axial symmetry such as very elongated impenetrable spheroids, as well as spheroidal elastic solids and shells.[6-11] The latter problem (elastic shells) is the most difficult of the problems mentioned within the context of the EBC method and might be more appropriately solved using approximate thin shell theories or Hackman's approach using spheroidal functions.[12] Weighted residual methods, the method of moments, both based on global and local basis states, are also showing progress toward solving a variety of problems particularly for complicated structures. Familiarity with the subject for the free-state problem and the many complications and numerical pitfalls that are encountered cannot help but lead one to the conclusion that when these objects are placed in a bounded environment, complications proliferate possibly to the point that the methods, if possible to formulate, are not numerically viable. It therefore seems at the very least judicious to attempt to construct any theory that describes what happens to a signal once it propagates from an object in a waveguide based on the free-state solution, in particular, one should try to determine a suitable unifying procedure that allows one to couple the free-state solution to a waveguide solution.

The purpose of this paper is to give a brief outline of a general method that allows one to produce the correct near-field solution that can be used in the development of a coupling scheme that will lead to a solution in a general

waveguide. Although the near-field solution is of inherent interest once it is obtained, there are several ways to couple the near-field solution to a waveguide and that has been discussed elsewhere. In this work we will emphasize primarily the near-field solution along with a new normal-mode theory designed to produce a general solution, but that can easily be converted to a spherical representation suitable for operating on irreducible tensors such as the T-matrix.

II. THE NEAR-FIELD PROBLEM

We wish to determine the near field due to the interaction of a guided wave with a submerged object. We can then use the near-field solution in a variety of strategies to obtain an approximate solution for the field scattered/ propagated from an object in a waveguide. Note that we do not state the problem as either a scatter or propagation problem because we can view the initial event (the interaction of the guided wave with the object) as a scattering event while we can view the course of the scattered signal as a propagation event once the waveguide takes effect. By employing the near field in this manner it is clear that we have divided the problem into different computational, as well as conceptional segments, namely propagation-scattering-propagation. In this section we show how to treat the interaction of a signal propagating in a waveguide with a submerged object. To do this, we first review the treatment of scattering from a target via the EBC method, which leads to a T-matrix which transforms an initial field into a scattered field (for the free-state problem) in matrix form as follows:

$$f=Ta \tag{1}$$

where f {a} are column vectors representing the expansion coefficients of the scattered (incident) fields, and T is the transformation (a square matrix) that maps the incident field onto the scattered field. Let us restrict our argument (for the sake of simplicity) to that of an axisymmetric target, such as a spheroid, and deal initially with a plane incident wave. We will then generalize to the case of a guided wave based on a normal-mode solution. We will see below that even for a fairly general range-dependent waveguide, this approach will be sufficient for calculation. To be specific, we look at the form of a T-matrix for a rigid axisymmetric target. To derive a T-matrix from the EBC equations of Waterman, one starts with the integral representation of the Helmholtz equation for a rigid target, namely:

$$U(\rho)=U_i(\rho)+\int_s [U_+(\rho')\partial G(\rho,\rho')/\partial n]dS \tag{2a}$$

and

$$0=U_i(\rho'')+\int_s [U_+(\rho')\partial G(\rho'',\rho')/\partial n]dS \tag{2b}$$

where $U(\rho)$, $U_i(\rho)$, $U_+(\rho')$ are the total field at a point ρ exterior to the object, the incident field, and the field at a point ρ' on the object, respectively, and $\partial G(\rho,\rho')/\partial n$ is the unit normal of the Green's function on the surface of the object. We are working with the velocity potential U throughout this development. The point r" is inside the bounded object and leads to a total field that is zero, hence the terminology "null field equation." Notice that we have two equations here, one representing the solution in the exterior and one the interior. There are two reasons for this. One is that we can employ the second of the two equations to eliminate the unknown in Eq. 2 and the second is that by including the interior solution we are able to avoid the spurious eigenvalues (irregular values) that arise when only the exterior solution is used. In Waterman's strategies, to solve this problem one must reduce the equations to that of a matrix problem by two devices. First one represents the Green's function in a biorthogonal expansion as follows:

$$G(\rho,\rho')=ik\Sigma Re\ \varphi_i(k\rho_<)\varphi_i(k\rho_>) \tag{3}$$

where Re $\varphi_i(\rho_<)$ and $\varphi_i(\rho_>)$ are the regular and outgoing partial wave functions. In the spherical polar representation $\varphi_i(k\rho)=h_i(k\rho)Y_{im}(\theta,\phi)$ where h_i and Y_{im} are outgoing spherical Hankel functions and spherical harmonics of order i, with m being the azimuthal index.

The next step in the solution to the problem is to expand the surface quantity in a partial wave or eigenexpansion, which is known to be complete on the surface of the object. There are many possible choices, but a judicious one for numerous reasons is the following:

$$U_+(\rho)=\Sigma_n b_n Re\varphi_n(\rho). \tag{4}$$

Here, b_n is now the unknown quantity in this relation that reduces the problem to that of finding the unknown set of numbers b_n, rather then the quantity U_+. This choice of the expansion guarantees that in the limit of a sphere, the EBC results agree formally with normal-mode theory. In addition, the regular spherical Bessel function $j_n(k\rho)$ offsets the contribution of the $h_n(k\rho)$ terms at least in the lower half of the matrix in the integrals that define the Q matrices (below) and thus avoids a problem associated with the dynamic range of the computer. (It is possible for the integral to be rapidly oscillatory with the difference in absolute value greater than the dynamic range of the computer. This can happen for some choices of expansion functions.) It is then possible (after suitable truncation) to arrive at the following set of matrix equations that relate the unknowns $\{f_n\}$ and $\{b_n\}$, where b is a column matrix representing the expansion coefficients of U_+.

$$f=ikReQb \tag{5a}$$

$$a=-ikQb. \tag{5b}$$

The transition matrix is then easily seen from Eq. 1 to be:

$$T=-ReQ\ Q^{-1} \tag{6}$$

where $Q_{ij}=\int Re\ \varphi_i(k\rho)\partial\varphi_i(k\rho)/\partial n\ dS$.

The T-matrix was developed in a spherical representation. It is a second rank tensor in irreducible form, and thus, only has meaning when operating on a vector also in a spherical representation. Since it is in irreducible form, it can be easily rotated once the simplest form of T is devised, which is a salient property. On the other hand, the fact that it requires the vector that it operates on to be in a spherical representation while having its value for plane waves (which are easily expanded in a spherical representation) impose restrictions on the form of the guided wave. However, we show in what follows how it is possible to represent a waveguide solution in a spherical representation of a fairly general form. The plane wave solution may be written in a partial wave representation via the Raleigh series as follows:

$$EXP\{i\rho k\cos(\theta)\}=\Sigma\ (2n+1)(i)^n\ j_n(\rho k)P_n(\theta) \tag{7}$$

where θ is the angle between k and ρ. For the more general representation one can use the addition theorem for spherical harmonics to obtain the plane wave in coordinates relative to a fixed Cartesian system, which we must ultimately use. That expression is:

$$P_n(\theta)=4\pi/(2n+1)\ \Sigma\Sigma\ Y_{mn}(\theta_{\kappa n},\ \varphi_{\kappa n})Y_{mn}(\theta_{\rho'n},\ \varphi_{\rho'n})^* \tag{8}$$

where θ is the angle between ρ and k and $\theta_{\eta n}$ and $\varphi_{\eta n}$ are the respective angles relative to the Cartesian coordinates of η for mode n.

Our interest, however, is in the guided wave impinging on a bounded object. Here the wave that insonifies the object for a stratified environment is of the form:

$$U_0=1/2(e^{i\pi/4})/d\Sigma\varphi_n(\gamma_n Zs)\ \varphi_n(\gamma_n Z)\ e^{\kappa n'}/(\kappa_n r)^{1/2} \tag{9}$$

where $\varphi_n(\gamma_n z)$ is the vertical solution to the problem, which is an eigenfunction with eigenvalue γ_n corresponding to the vertical wave number for mode n, while z_s and z are the values of vertical displacement at the source and at observation respectively. r is the distance in the horizontal plane and κ_n is the horizontal wave number for mode n. Here $k^2 = \gamma_n^2 + \kappa_n^2$. In this formulation, we include only propagating modes and assume that the origin of the source is localized in space such that it may be approximated by a point source and that we are sufficiently far from the source that evernacent modes can be ignored. This procedure is, in fact, standard practice in the derivation of normal-mode equations and proves to be quite adequate for many environments. We will limit discussion to layered fluid bottoms so that U symbolizes the velocity potential. For an elastic bottom we would have to formulate the problem using the displacement vector in order to satisfy the boundary conditions at the fluid-elastic interface.

It would be instructive at this point to illustrate with an example how to convert U to a spherical representation for the case in which we represent the environment by n isovelocity layers. Let us pick a particular layer for which the submerged object resides. Then U is of the form:

$$U_o = 1/2(e^{i\pi/4})/d\Sigma a_n \sin(\gamma_n z_s) \sin(\gamma_n z) e^{\kappa_n r}/(\kappa_n r)^{1/2} \tag{10}$$

where a_n is the expansion coefficient. We want to take advantage of the Rayleigh expansion of a plane wave so we write

$$\sin(\gamma_n z) e^{\kappa_n r} = \{\exp(i\,\gamma_n z + \kappa_n r) - \exp(-i\gamma_n z + \kappa_n r)\}\,/2i \tag{11}$$

where the (x,y,z), (r,z) are illustrated in Fig. 1. The coordinates (x',y,z') for the T-matrix must *preforce* be in the representation in Fig. 2 where z' is along the axis of symmetry of the spheroid (our choice of target). We write $\cos(\alpha_{n+}) = (\gamma_n z + \kappa_n r)/_{k\rho}$ and $\cos(\alpha_{n-}) = (\gamma_n z - \kappa_n r)/_{k\rho}$ so that we have

$$\sin(\gamma_n z) e^{\kappa_n r} = \{\exp[i\,k\rho\,\cos(\alpha_{n+})] - \exp[i\,k\rho\,\cos(\alpha_{n-})]\}\,/2i\,. \tag{12}$$

Thus, after use of the Rayleigh series and the above expression, we have U in the following spherical representation:

$$U_o = 1/2(e^{i\pi/4})/d\Sigma_n\Sigma_L a_n(2L+1)(i)^L j_L(\rho k)[P_L(\alpha_n) - P_L(\alpha_{n-})]/(\kappa_n r)^{1/2} \tag{13}$$

where we must use the spherical harmonic addition theorem (Eq. 8) to obtain the most general form of the expression. The above expression has been derived tacitly assuming that the interaction between the guided wave and the submerged object occurs at a point. In fact, the interaction is extended in space and we must allow for this in the final development. The final form depends on the way we interface the object with the guided wave. Let us assume we have derived the above expression to be valid at the origin of the submerged object. Further, we wish to find the field at some vertical line or over some surface with distance r_0 from the center of the object. This will be developed later. However, in the next section we introduce a more general T-matrix, one that includes objects near an ocean surface.

III. AN ENHANCED T-MATRIX FOR OBJECTS NEAR AN OCEAN SURFACE

So far we have only dealt with the free-state T-matrix. If the object is near a surface, then we have ignored multiple interactions between the object and the interface. Although, for most problems this multiple interaction will produce high angle propagation that will ultimately get absorbed into a realistic (attenuating) bottom and should not be a strong factor in calculation. We show how this effect can be included in our formulation for completeness. Previously we have shown how it is possible to derive a T-matrix for scattering near an interface. It takes the form for only one impenetrable interface:

$$T_{half} = R(d)T\{(1-\sigma(-2d)T\sigma(2d)T)^{-1}[R(-d)+\sigma(-2d)TR(d)]+R(-d)T$$

$$X\{(1-\sigma(2d)T\sigma(-2d)T)^{-1}[R(d)+\sigma(2d)TR(-d)] \tag{14}$$

where R(d) and σ(d) are translation matrices that translate spherical Bessel and Hankel functions by the distance d (in this case from the surface of the ocean). The translation/rotation properties of the spherical Bessel functions $\{j_n(kr)\}$ are easy to derive by noting that $\exp\{ik(d+r)\cos(\alpha')\}= \exp\{ikr\cos(\alpha)\}\exp\{ikr\cos(\alpha'')\}$ where the α's are the angles between k and (d+r), r, and d, respectively. Then by expanding each exponential in the Rayleigh series and using the addition theorem for spherical harmonics, multiply the left and right series by the complex conjugate of the spherical harmonics associated with the left hand side and integrate overall space using orthogonality of the spherical harmonics to obtain a series on the right equal to $j_n[k(d+r)]$. The two integrals are well known to be related to Clebsch-Gordan (C-G) coefficients so that the integrals may be replaced by the C(ll'l", m, 0, m'). We use the more symmetric 3-j form of these overlap functions represented to be doubled column brackets to obtain:

$$jn[k(d+r)]=\Sigma R_{nn'}(kd)j_{n'}(kr) \tag{15}$$

where for translation along the z-axis we have:

$$R(\pm d)_{mnm'n'} = \sum_{\lambda=|n-n'|}^{n+n'} (-1)^{m+n+(n'+m'+\lambda)/2} (2\lambda+1)\sqrt{(2n+1)(2n'+1)}$$

$$\begin{pmatrix} n\,n'\,\lambda \\ 0\,0\,0 \end{pmatrix}\begin{pmatrix} n\,n'\,\lambda \\ m\,m\,0 \end{pmatrix}(\pm 1)^{\lambda}\,j_{\lambda}(kd)\delta_{mm} \tag{16}$$

where $\begin{pmatrix} n\,n'\,\lambda \\ m\,m\,0 \end{pmatrix}$ are 3j – coefficients that are related to the C-G coefficients by simple factors.

This relation defines the R's, and if we replace the j's by h's then it also defines the σ's. It is easy to see from the exponential nature of the Rayleigh series that R(d)R(–d)=1 and R(d)R(d)=R(2d). The definition of σ(d) is more difficult to derive but along with R(d) the two expressions are defined by the one equation above. It can be shown easily that R and σ are unitary so that for example σ(d)'*= σ(–d). Unfortunately, this sort of translation is only suitable for a sphere or an object in which the body z-axis (the axial direction) is aligned with the z-direction of the waveguide. This is seldom the case. In fact, it is usually perpendicular with the z-axis. In that case the R's are not diagonal with respect to the m's. In this form, the translation matrices R and σ are not independent of the azimuthal index m (which for example occurs in the spherical harmonic above). It is to be noted that for axisymmetric objects the free space T-matrix and the Q-matrices are diagonal in m (they are block diagonal, i.e., of the Jacobi form). Thus, we can invert the Q's for each m. Had that not been the case, we would have to invert the Q's for matrix elements including mixed m's (this occurs for general targets) and this would reduce the combativeness of the T-matrix method except for low frequencies. Because waveguides are not axisymmetric in range, we expect (in general) to couple the m's and in fact the presence of the translation matrices in expressions of the form $(1-\sigma(-2d)T\sigma(2d)T)^{-1}$ renders the evaluation numerically unviable except for the special case mentioned above. However, we can expand the expression as follows $(1-\sigma(-2d)T\sigma(2d)T)^{-1}=1+\sigma(-2d)T\sigma(2d)T+\sigma(-2d)T\sigma(2d)T\sigma(-2d)T\sigma(2d)T+....$

Since σ(–2d) approaches small values with increasing d, it is not difficult to suppose that this series terminates fairly quickly. Further, since each term corresponds to zero, one, two..., etc., reflections from the surface then on physical grounds one expects convergence quickly. Indeed, if one has n multiple reflections, then it can be shown that the corresponding expansion term dies off as $1/(2d)^{2n}$. By retaining only a few terms corresponding to only a few multiple interactions between the surface and the object, it is then viable to include multiple reflections in the way just prescribed. We will not include this effect here.

IV. A NEW NORMAL-MODE METHOD SUITABLE FOR THE NEAR-FIELD PROBLEM

A. A New Full-Perturbation Method

In general the normal-mode solution involves a water column that has a variable velocity. In the above development we assume an isovelocity environment. We could divide the variable velocity profile into isovelocity layers and treat the target as being in one of the isosegments and proceed from there. This can lead in some cases to

complications and so we have developed a new normal-mode method that obviates this problem and also leads to a more elegant formulation. Because of the importance of this problem, we will outline the method here. The method is a full perturbation approach that obtains solutions to the vertical component of the normal-mode problem. This method proves to be very fast and particularly suitable for including fluctuations in a waveguide, as well as coupling schemes for including targets in the waveguide. We write the exact solution for an unperturbed case as follows:

$$\frac{d^2\Psi_i}{dz^2} + \left[k_0^2 - \lambda_i^0 \right]\Psi_i = 0. \tag{17}$$

The desired solution is rewritten as follows:

$$\frac{d^2 U_i}{dz^2} + \left[k^2(z) - \lambda_i \right]U_i = 0. \tag{18}$$

For convenience we rewrite Eq. 18 as follows:

$$\frac{d^2 U_i}{dz^2} + \left[k_0^2 - \lambda_i^0 \right]U_i = QU_i \tag{19}$$

$Q = k_0^2 - k^2(z) + \lambda_i - \lambda_i^0 = q + \Delta\lambda_i$ and $q(z) = k_0^2 - k^2(z)$ and $\Delta\lambda_i = \lambda_i - \lambda_i^0$.

For the isovelocity case we can rewrite Eq. 17 as: $\dfrac{d^2\Psi_i}{dz^2} + \alpha_i^2 \Psi_i = 0$ where $\alpha_i^2 = k_0^2 - \lambda_i^0$.

We impose the following orthonormality conditions: $\left(\Psi_i, \frac{1}{\rho}\Psi_j \right) = \delta_{ij}$ and $\left(U_i, \frac{1}{\rho}U_j \right) = \delta_{ij}$.

We assume closure so that we can express U_i:

$$U_i = \sum_{j=1}^{N} a_{ij} \Psi_j. \tag{20}$$

We insert Eq. 19 into Eq. 20 to arrive at:

$$\sum_{j=1}^{N} a_{ij}\left(\alpha_i^2 - \alpha_j^2 \right)\Psi_j = \sum_{j=1}^{N} a_{ij}\left(q(z) + \Delta\lambda_i \right)\Psi_j \tag{21}$$

where $i = 1, 2, 3 \ldots N$.

By integrating the overlap of the above expression with Ψ_i we obtain:

$$\lambda_i = \lambda_i^0 - q_{ii} - \sum_{i \neq j}^{N} \bar{a}_{ij}q_{ij} \text{ where } \bar{a}_{ij} = \frac{a_{ij}}{a_{ii}}. \tag{22}$$

This yields the eigenvalue correction equation.

By integrating the overlap of Eq. 21 with Ψ_k we obtain: $\tilde{a}_{ik}\left(\alpha_i^2 - \alpha_k^2 - \Delta\lambda_i\right) - \sum\limits_{j \neq i}^{N} q_{jk}\,\tilde{a}_{ij} = q_{ik}$ where $k = 1,$ 2, 3 . . . N and $i \neq k$.

This is an eigenvalue problem. We can use the expansion for the eigenvalue above to rewrite the equation as follows: $\tilde{a}_{ik}\left(\alpha_i^2 - \alpha_k^2 + H_{ik}\right) - \sum\limits_{j \neq \{i,k\}}^{N} q_{jk}\,\tilde{a}_{ij} = q_{ik}$ where $k = 1, 2, 3$ . . . N and $i \neq k$.

The diagonal terms are $\left(\alpha_i^2 - \alpha_k^2 + H_{ik}\right)$. Where $H_{ik} = q_{kk} - \Delta\lambda_i$.

This will prove useful later. The H_{ik} terms contain the first and the higher order terms and in many cases part of the terms are negligible. The diagonal terms are almost always greater then the off-diagonal terms. Thus, we may in some cases use the Gauss-Seidel method. The first iteration via Gauss-Seidel leads to:
$\tilde{a}_{ik} = \dfrac{q_{ik}}{\alpha_i^2 - \alpha_k^2 + q_{kk} - \Delta\lambda_i} = \dfrac{q_{ik}}{\alpha_i^2 - \alpha_k^2 - H_{ik}}$, where we rewrite $H_{ik} = q_{kk} - q_{ii} - \sum\limits_{j \neq i}^{N} a_{ij}\,q_{ij}$.

This can be a big improvement over the ordinary perturbation term in that $H_{ik} > q_{ik}$, and thus, we expect that generally we will have convergence if the master matrix is diagonally strong. We see readily that we can arrive at two improved perturbation theories by retaining the full form of H_{ik} or by excluding the last term (the higher order terms) of H_{ik}. The advantage to excluding the last term is that we need not employ a sophisticated perturbation approach to be discussed below. Further, this expression shows that the old theory overestimated the expansion coefficient.

The new complete perturbation expansion for the eigenvalue is now: $\lambda_i = \lambda_i^0 - q_{ii} - \sum\limits_{i \neq j}^{N} \dfrac{q_{ij}q_{ji}}{\alpha_i^2 - \alpha_j^2 + H_{ij}}$.

This expression indicates that the first order correction is still the same, but the higher order corrections are over/under estimated in the old theory. We must now contend with the eigenvalues which turns out to be fairly easy to deal with.

B. A Solution to the Eigenvalue Problem

We can reorder the above equations selectively to arrive at: $Qa = \lambda a$, where $Q_{ij} = -q_{ij}$ $i \neq j$ and $Q_{ii} = \lambda_i^0 - q_{ij}$.

Thus, we have an eigenvalue problem for the λ's. In the above expression the matrix Q is real, symmetric, and diagonally strong. A rather good strategy to solve this eigenvalue problem is to make use of the Householder method to reduce Q to tridiagonal form. For an $n \times n$ matrix this only takes $n-2$ orthogonal transformations. Since the eigenvalues of this very stable and fast reduction method are unchanged, we need only find the eigenvalues of a tridiagonal matrix that can be done in n operations. The quantity a in the above expression is rotational (unitary for complex eigenvalues) and ought to correspond to the expansion coefficients required in reconstructing the exact eigenvalues. Indeed, that fact that the matrix a is rotational or unitary under certain conditions guarantees that the newly constructed eigenvalues are orthogonal as is required for Sturm-Liouville problems. We use the above method just to obtain the eigenvalues and revert to the master equation that defines a along with H_{ik} to obtain the expansion coefficients. This seems to work well. We now have a method that allows us to go to at least 10,000 Hz in shallow water, a value considerably above what is usually allowed in normal-mode techniques, and at a considerable improvement in speed. We now have exactly the type of method that allows one to expand a general normal-mode solution in terms of a spherical representation because each normal mode is in terms of a sin function which when included with the solution in range can easily be written in a spherical representation as was done in the earlier section for the isovelocity case.

V. THE NEAR FIELD

We briefly outline the basis for the method used to describe scattering from an object in a waveguide in the near field based on the preceding results. We begin by allowing a guided wave to impinge on an object as it traverses a given region. The object scatters the guided wave in some manner. We determine the scattered field near the object via a free-field transition matrix as described above. Let us choose a surface circumscribing the submerged object. The surface is arbitrary to the extent that it is smooth and its dimensions do not exceed the boundaries of the waveguide and should be such that the largest dimension of the surface is such that the highest angle mode does not interact with the boundary of the circumscribed region. We next require the near field on the surface, as well as its normal derivative. To do this we must make a transformation from the coordinates of the waveguide to that of one relative to the axis appropriate to the submerged object. Recalling that the mode angles α_n were obtained above relative to the horizontal (and not the vertical as is usual in normal-mode theory), we choose β to designate the angle that the mode makes with the axis of symmetry of the spheroid in the horizontal plane. It is to be noted that the reference axis of the spheroid and that of the waveguide differ. In particular, in order to exploit the axial symmetry of the object, we must choose the z-axis in the object body reference along the axis, its axis of symmetry, while we can choose x and y at our convenience. In the waveguide, z is in the downward direction while x and y are in the horizontal plane. The angle scheme chosen is indicated in Fig. 2, where θ_n and φ_n are the angles for the particular mode n the appropriate angles to be implemented in the spherical harmonics. They are, in particular, the angles of the incident mode n relative to the symmetry axis of the submerged object, the angle that the plane generated by the incident raymode n, and the symmetry axis the spheroid makes above (below) the horizon, respectively. They are:

$$\tan(\varphi_n)=\tan(\alpha_n)/\sin(\beta) \tag{24a}$$

$$\cos(\theta_n)=\cos(\beta)\cos(\alpha_n) \tag{24b}$$

where the surface is chosen at a suitable region circumscribing the object (suitably near the object) with origin at the center of the object, this can be obtained from the expression

$$f(\rho,\theta,\varphi)=\Sigma\Sigma\alpha_{mm'nn'}T_{mm'nn'}(\kappa_n) \ Y_{mn}(\theta_{\kappa m'n}, \varphi_{\kappa n})Y_{mn}(\theta_{r'nm'},\varphi_{r'n}) \ h(\kappa_n\rho)/(\kappa_n\rho)^{1/2} \tag{25}$$

where the α's are projection coefficients of the normal-mode functions onto the spherical (partial wave) solutions. This is the most general form of the scattered near field.

VI. FREE-FIELD CALCULATIONS

There are two classes of targets for impenetrable problems, i.e., soft and hard scatterers. Soft acoustically-impenetrable targets do not support body resonances; therefore, we examine acoustic quantities appropriate for nonresonant targets. These quantities are bistatic and monostatic angular distributions, defined by the case of noncoincident source and receivers, where receivers vary through all possible rotation angles. Angular distributions are thus dependent on target geometry and can be useful to determine such features of target shape as symmetry or elongation. In particular, reflection, diffraction, and generalized Snell's law behaviors can be observed as curved-surface analogs for the plane-layered case.

Bistatic angular distributions correspond to measurement of a scattered field at any point in space for some incident wavefield fixed relative to some source-object orientation. In Fig. 3 we examine a rigid spheroid of aspect (length-to-width) ratio of 15:1. Fig. 3a–d represent scattering from the object along the axis of symmetry (end-on) (a) 30, (b) 60, (c) 90, and (d) degrees relative to the symmetry axis (broadside). The values of the incident wavefield frequency are expressed using the dimensionless quantity $kL/2$, where L is the object length and k the total wavenumber $(k=2\pi/\lambda)$. The value of $kL/2$ in Fig. 3 is 30, which implies that the object is about 10 wavelengths long and thus in the intermediate-frequency region where neither low nor high frequency approximations apply. In all figures, frequency is sufficiently high that wave diffraction effects are significant in the forward scattering direction. Perhaps the most interesting feature of the four plots (Fig. 3a–d) corresponds to a reflection at the (fairly flat) side of the

4. P. C. Waterman, "Matrix theory of elastic wave scattering," *J. Acoust. Soc. Am.*, 60(3), 567, 1976.

5. P. C. Waterman, "Matrix theory of elastic wave scattering II: a new conservation law," *J. Acoust. Soc. Am.*, 63(6), 1320, 1977.

6. A. Bostrom, "Scattering of stationary acoustic waves by an elastic obstacle immersed in a fluid," *J. Acoust. Soc. Am.*, 67(2), 390, 1980.

7. B. A. Peterson, V. V. Varadan, and V. K. Varadan, "T-matrix approach: study of vibrational frequencies of elastic bodies in fluids," *J. Acoust. Soc. Am.*, 74(5), 1051, 1983.

8. M. F. Werby and L. R. Green, "An extended unitary approach for acoustical scattering from elastic shells immersed in fluids," *J. Acoust. Soc. Am.*, 74(2), 625, 1983.

9. M. F. Werby and S. Chin-Bing, "Numerical techniques and their use in extension of T-matrix and null-field approaches to scattering," *Int. J. Comp. Math. Appls.*, 11(7/8), 717, 1985.

10. M. F. Werby, "A coupled higher-order T-matrix," *J. Acoust. Soc. Am.*, submitted, 1989.

11. M. F. Werby and R. D. Evans, "Scattering from unbounded and bounded objects," *IEEE J. Ocean. Eng.*, OE-12, 380–395, 1987.

12. R. H. Hackman and G. S. Sammelmann, "Acoustic scattering in an inhomogeneous waveguide," *J. Acoust. Soc. Am.*, 80(5), 1447, 1986.

13. J. S. Cohen and B. F. Cole, "Shallow water propagation under downward-refraction condition II," *J. Acoust. Soc. Am.*, 61, 213–217, 1977.

Table 1. Waveguide Characteristics

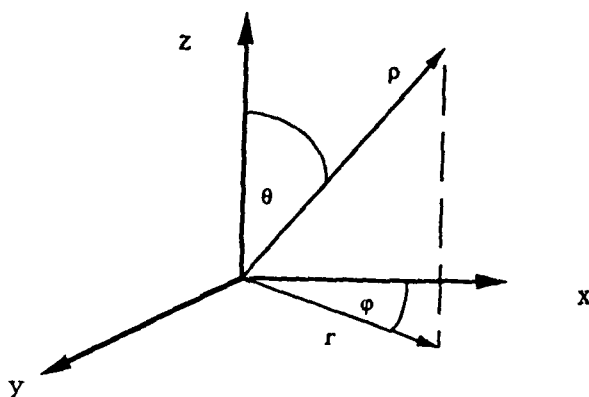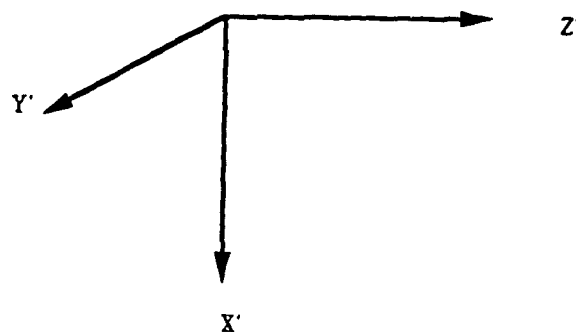| WATER DEPTH (m)	SOUND SPEED (m/s)	DENSITY (g/cc)
0.0	1449.0	1.0
180.0–	1440.0	1.0
180.0+	1530.0	2.0

Fig. 1.

Fig. 2.

object for scattering angles of 30 and 60 degrees. This reflection can occur only for very elongated objects that approach flat surfaces, so that the reflected angle is almost the same as the incident angle (relative to a straight line through the axis of symmetry).

We now consider the same scattering problem for sound-soft objects. Fig. 4a–d illustrates the response from a spheroid of 15:1 aspect ratio and a kL/2 of 15. In terms of directionality and reflection, these angular response patterns are similar to those in the previous rigid case. The scattered field tends toward the forward direction and the reflected angles (relative to the axis of symmetry) coincided with the incident wavefield. However, scattered fields are now more highly focused, even at the lower frequency values considered. This latter effect is due to the phase-change at the object surface (π radians). We see the same effect (as for the rigid case) in b and c for surface reflections at angles equal to the incident value relative to the axis of symmetry.

VII. NEAR FIELD IN A WAVEGUIDE

Because the vertical component of an ocean waveguide normally varies slowly, it may be approximated by many horizontally stratified water layers overlaying many horizontally stratified sediment layers, bounded below by a high speed, strongly absorbing, half-space, and above by a pressure release surface. Each water and sediment layer is defined by its local sound speed and density. The radiation condition in the half-space causes the field to diminish exponentially with increasing depth. The waveguide is assumed to have azimuthal symmetry. For this example, shear is ignored in the sediment and the half-space.

The purpose of the following example is to examine the backscatter signal from a solid rigid spheroid situated in a realistic shallow-water environment, after being insonified with a continuous wave (CW) source with a frequency of 100 Hz. The environment selected was a shallow-water waveguide representing the coastal region off the North east coast of the U.S. in May. Its parameters are shown in Table 1. The environmental description was developed by Cohen and Cole.[13] A solid rigid spheroid 50 m long and 10 m in diameter (aspect ratio 5:1) was placed 5 km from the source. The source and receiver depths were at 90 m, while the object was at 50 m. The source, receiver, and object, in general, can be at different depths in the waveguide. Eight modes were excited.

Figure 5 represents a contour plot of the scattered field that resulted when the object was insonified by the source at an angle of 45 degrees from the front of the object. The scattered field was calculated every 10 m in range starting at 4500 m and ending at +500 m. Note that the field is expressed in terms of transmission loss in decibels relative to one meter. The calculations were not carried out when the receiver was closer than the mathematical surface circumscribing the object and the value was arbitrarily set to 200 dB, hence the circular opening of the field. The kL/2 value for this example was only 10.48 which explains the broad lobes shown in the figure. Note also that in the forward and specular direction the field is strong, approximately 116 dB. The field is very weak along the major axis of symmetry, approximately 140 to 170 dB. This is consistent with observations made in free space. Figure 6 is the same field depicted as a mesh plot with the contour plot of Fig. 5 overlaid. This depiction better shows the effect of the waveguide upon the scattered field.

VIII. ACKNOWLEDGMENTS

IX. REFERENCES

1. G. V. Norton, M. F. Werby, and E. White, "Influence of a waveguide on backscattered signals in the far field." SPIE Proceedings, 1993.

2. P. C. Waterman, "New foundation of acoustic scattering," *J. Acoust. Soc. Am.*, 45, 1417, 1969.

3. P. C. Waterman, "Matrix formulation of electromagnetic scattering," *Proc. IEEE*, 53(3), 802, 1965.
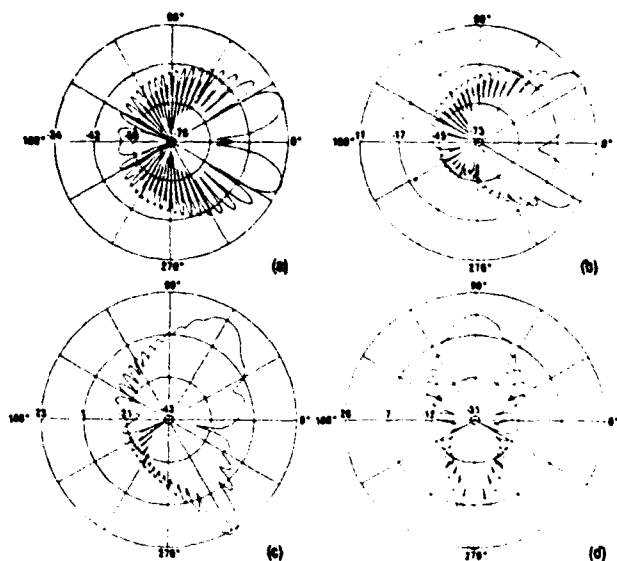
Fig. 3. Numerical examples (a) scattering from a rigid spheroid, aspect ratio 15:1, $kL/2 = 30$, along symmetry axis; (b) scattering from a rigid spheroid, aspect ratio 15:1, $kL/2 = 30$, 30 degrees relative to symmetry axis; (c) scattering from a rigid spheroid, aspect ratio 15:1, $kL/2 = 30$, 60 degrees relative to symmetry axis; (d) scattering from a rigid spheroid, aspect ratio 15:1, $kL/2 = 30$, 90 degrees relative to symmetry axis.
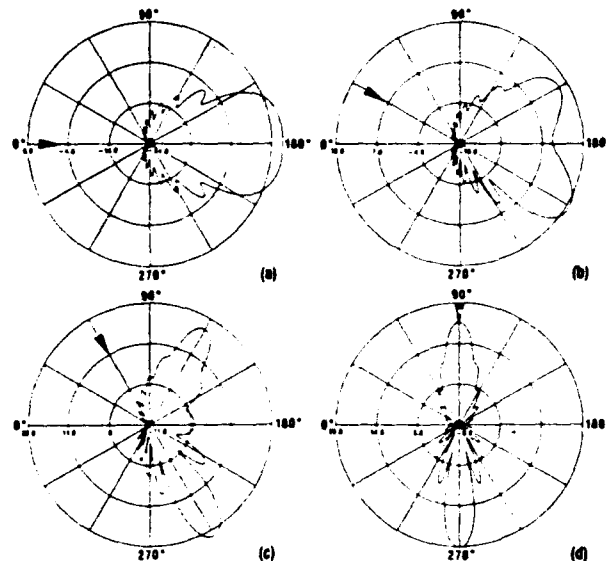

Fig. 4. Numerical examples (a) scattering from a soft spheroid, aspect ratio 15:1, $kL/2 = 15$, along symmetry axis; (b) scattering from a soft spheroid, aspect ratio 15:1, $kL/2 = 15$, 30 degrees relative to symmetry axis; (c) scattering from a soft spheroid, aspect ratio 15:1, $kL/2 = 15$, 60 degrees relative to symmetry axis; (d) scattering from a soft spheroid, aspect ratio 15:1, $kL/2 = 15$, 90 degrees relative to symmetry axis.
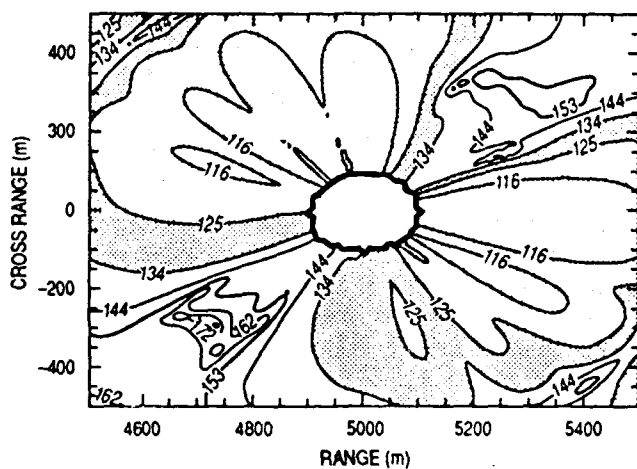

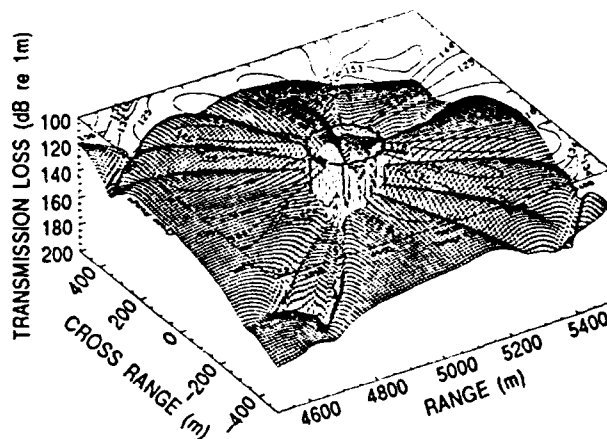Fig. 5. Contour plot of the scattered field from an object in a waveguide.

Fig. 6. Mesh plot of the scattered field with an overlay of Fig. 5.